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Decay of unstable D-branes with electric field

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Abstract: Using the techniques of two dimensional conformal field theory we construct time dependent classical solutions in open string theory describing the decay of an unstable D-brane in the presence of background electric field, and explicitly evaluate the time dependence of the energy momentum tensor and the fundamental string charge density associated with this solution. The final decay product can be interpreted as a combination of stretched fundamental strings and tachyon matter.

Keywords: Tachyon Condensation, D-branes.

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Contents

1. Introduction and summary

It has been noticed in various works [1]–[6] that switching on an electric field on the world-volume of a D-brane leads to various new physical effects which are absent in the magnetic case [1, 7, 8]. In particular, unlike the magnetic case the strength of the electric field can not be increased beyond a critical value keeping the open string theory stable. The solutions carrying electric flux [9]–[12] are particularly interesting in the tachyon vacuum of an unstable D-brane [13, 14], since in this vacuum the full Poincare invariance of the bulk is expected to be restored and all the perturbative degrees of freedom are unphysical [15]. It was shown in [11] that although the effective action proposed in [16] vanishes at the tachyon vacuum, the system admits a well-defined hamiltonian description, and has classical solutions describing electric flux tubes carrying fundamental string charge. The classical dynamics of these flux tubes is described by Nambu-Goto action, and possesses the full Poincare invariance of the bulk [11, 12].

A related development in the study of tachyon dynamics has been the construction of time dependent classical solutions representing the decay of an unstable D-brane as the tachyon rolls down towards the minimum of the potential [17]–[21].[1] The solution

[1] See [22] for the study of time dependent solutions representing rolling of tachyons in p-adic string theory, ref. [23] for a time dependent solution in cubic string field theory, [24] for related cosmological applications and [25] for other related works including discussions in the context of background independent string field theory. Earlier attempts at cosmology involving rolling tachyon were made in [26].

JHEP11(2002)047

is constructed by perturbing the boundary conformal field theory (BCFT) describing the original D-brane by an exactly marginal deformation. The strength $\widetilde{\lambda}$ of the perturbation labels the initial value of the tachyon T. The perturbed BCFT carries all the information about this one parameter family of rolling tachyon solutions labelled by the initial position of T. For example the corresponding boundary state gives information about the time evolution of various closed string sources, e.g. the energy-momentum tensor. In particular, it was shown in [19] that if we displace the tachyon towards the (local) minimum of the potential and let it roll, the system asymptotically evolves to a pressure-less gas with non-zero energy density confined in the initial D-brane world-volume. It was also shown that at $\widetilde{\lambda} = 1/2$, which corresponds to placing the tachyon at the minimum of the potential, the energy-momentum tensor vanishes at all time as conjectured [13, 14].[2]

Given these results, we can ask: how does the energy momentum tensor (and other closed string sources) evolve with time for the rolling tachyon solution in the presence of electric flux? In this paper we answer this question by constructing a two parameter family of time dependent solutions, labelled by the initial electric field e and the initial value $\widetilde{\lambda}$ of the tachyon. We find, first of all, that the solution carries fundamental string charge as expected. At arbitrary values of $\widetilde{\lambda}$ and the electric field e (below the critical limit), the energy-momentum tensor $T_{\mu\nu}$ of the solution splits into a sum of contribution from two sources, — the rolling tachyon, and a uniform density of fundamental strings [28], localised on the initial location of the brane, and stretched along the direction of the electric field. The contribution to $T_{\mu\nu}$ from the fundamental string charge remains constant in time, whereas the rolling tachyon contribution has the same form as that in the absence of electric field, except for a change in the overall normalisation and a time dilation which depends on the strength of the electric field. Since the time evolution of the rolling tachyon contribution gets affected by the value of the electric field, it shows that the two systems are coupled together. In a suitable limit where the initial value of the tachyon approaches the minimum of the potential ($\widetilde{\lambda} \to 1/2$) and the background electric field goes to its critical value ($e \to 1$), the contribution from the rolling tachyon drops out, leaving behind only the time independent fundamental string contribution.[3] This coincides with the source terms discussed in [28]. However our analysis also shows that this source has divergent components for the higher massive closed string states. The physical significance of this divergence has not been investigated in this work. However, since fundamental string configurations are valid configurations in string theory, our results can be turned around to conclude that divergent higher level contribution to a boundary state does not necessarily imply a singularity of the configuration that it describes. (Different aspects of higher level contribution to the boundary state associated with rolling tachyon configuration have been discussed in [29].)

We carry out our analysis in two different ways. In the first approach discussed in section 2 we assume the existence of a space-time effective field theory for the tachyon, possibly containing infinite number of higher derivative terms, and use the results of [8] to

[2]For early studies of open string tachyon dynamics, see [27].

[3]This implies that the electric flux solution obtained this way is a stationary solution.

write the sources for the massless closed string fields in presence of the background electric field in terms of the sources in absence of the background electric field. This does not require knowledge of the explicit form of the effective action, and is done by the following steps:

1. Using the results of [8] we first rewrite the action at a nonzero background electric field e using the open string metric and the non-commutative $*$ product. This relates a solution in the presence of background electric field to a solution in the non-commutative theory with zero background electric field. The dependence of the solution on e in the non-commutative theory comes only through the $*$ product and the open string metric.

2. Now if we look for a solution which depends only on one space-time direction in the non-commutative theory with zero background electric field, then one can immediately relate this to a solution in commutative theory with zero background field, since non-commutativity does not play any role if the configuration depends on only one direction. This allows us to relate a solution in the non-commutative theory, and hence in the commutative theory with background electric field e, to a solution in the commutative theory with vanishing background electric field. In the latter theory, the dependence of the solution on e comes from the dependence of the open string metric on e.

3. The solution in commutative theory with zero background field and non-trivial (constant) open string metric can be further related to the solution in trivial open string metric background by a linear coordinate transformation that takes the open string metric to the trivial metric.

This allows us to construct a solution in the theory with background electric field and trivial metric in terms of a solution in the theory with zero background electric field and trivial metric. The sources, evaluated at this pair of solutions in the two different theories can also be related by using their tensorial property under this coordinate transformation. Using this and the chain rule of differentiation one can then write down the sources derived from the lagrangian with a non-zero background electric field and trivial metric in terms of the sources derived from the lagrangian with zero background electric field and trivial metric. Since the sources in absence of the background field are already known [19], this gives the sources in the presence of the background electric field.

Our second approach discussed in section 3 is to directly analyse the BCFT corresponding to the rolling tachyon solution coupled to the background electric field. We construct this BCFT by adding appropriate perturbation to the BCFT describing the unstable D-brane with a constant background electric field. In [18], the perturbation associated with the rolling tachyon solution was identified from the spatially homogeneous solution to the linearised equation of motion for the tachyon in string field theory. Proceeding along the same line, we first find the solution to the linearised equation of motion in presence of the background electric field, and use it to identify the rolling tachyon perturbation in presence of background field. Having identified the relevant BCFT we analyse it following the approach of [17, 18], namely Wick-rotate the time direction to go to a theory with all spatial directions. The resulting theory without the perturbation term is equivalent to the standard magnetic BCFT analysed in [1, 2, 30, 8] with an imaginary magnetic field. We do all the explicit analysis with a real magnetic field and at the end recover results in the original electric theory by performing the inverse Wick-rotation and an analytic continuation which makes the magnetic field imaginary. We study the perturbation describing the rolling tachyon in this magnetic BCFT and argue, using the idea of *locality* of boundary operators introduced in [32], that this deformation is exactly marginal. This gives a two parameter family of BCFT's labelled by the background electric field e and the strength $\widetilde{\lambda}$ of the deformation. We then construct the boundary state associated with these BCFT's by generalising the construction given in [30]–[33] to the present case. Once the boundary state is known, we can extract the massless closed string sources from this boundary state [34, 35, 36]. These reproduce the results obtained through the target space analysis of section 2. The new information that one gets from the boundary state analysis is the divergence of the boundary state for the higher level terms in the limit $\widetilde{\lambda} \to 1/2$, $e \to 1$ discussed earlier. The divergence is demonstrated by performing an explicit computation of the terms at the next higher level.

All the above discussions have been made in the case of bosonic string theory. We generalise the analysis to the superstrings in section 4. We have computed only the massless sources by generalising the analysis of [19]. Although we have not computed the full boundary state or performed any computation at the next higher level, one might expect to see similar divergences in the $\widetilde{\lambda} \to 1/2$, $e \to 1$ limit in this case also.

In section 5 we discuss a candidate low energy world-volume effective action following [16, 37, 38, 19, 20] which reproduces the results for the sources of massless closed string fields. We also discuss its coupling to the supergravity fields including the massless RR backgrounds.

2. Target space analysis

We denote by $\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu})$ the effective action describing the dynamics of the tachyon field on a Dp-brane in the presence of constant background closed string metric $g_{\mu\nu}$ and anti-symmetric tensor field $b_{\mu\nu}$. By the result of [8], this effective action is equivalent to another action:[4]

$$\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu}) = \sqrt{\frac{\det(g+b)}{\det G}}\, \mathcal{S}_\Theta(T; G_{\mu\nu}, b_{\mu\nu} = 0)\,, \tag{2.1}$$

where $\mathcal{S}_\Theta$ is related to $\mathcal{S}$ by the replacement of all products by $*$-products with non-commutativity parameter Θ, and $G_{\mu\nu}$ is the open string metric. The inverse $G^{\mu\nu}$ of $G_{\mu\nu}$ and $\Theta^{\mu\nu}$ are given in terms of $g_{\mu\nu}$ and $b_{\mu\nu}$ by the relations:

$$G^{\mu\nu} = \left[(g+b)^{-1}\right]_S^{\mu\nu}\,, \qquad \Theta^{\mu\nu} = \left[(g+b)^{-1}\right]_A^{\mu\nu}\,, \tag{2.2}$$

[4]We have absorbed factors of 2π into the definition of $b_{\mu\nu}$ and $\Theta_{\mu\nu}$ compared to [8] in order to simplify various formulæ.

where the subscripts S and A denote the symmetric and anti-symmetric parts of the corresponding matrices respectively. In general the fields T appearing on the two sides of (2.1) are related by a field redefinition, but for the configurations we shall be considering where the tachyon depends only on the time coordinate, this field redefinition is identity.

The equivalence (2.1) implies that if we can construct a classical solution of the equations of motion of $\mathcal{S}_\Theta(T; G_{\mu\nu}, b_{\mu\nu} = 0)$, then we also have a solution of the equations of motion of $\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu})$. We shall focus on solutions which depend on only the time coordinate; and in this case we can replace the $*$-product by ordinary product. Thus we need to find solutions of the equations of motion of $\mathcal{S}(T; G_{\mu\nu}, b_{\mu\nu} = 0)$. For constant $G_{\mu\nu}$, these solutions, in turn, can be computed from solutions of the equations of motion of $\mathcal{S}(T; \eta_{\mu\nu}, b_{\mu\nu} = 0)$ by a linear transformation on the coordinates that converts the metric $\eta_{\mu\nu}$ to $G_{\mu\nu}$.[5]

To be more specific, we shall consider a background of the form:

$$g_{\mu\nu} = \eta_{\mu\nu}\,, \qquad b_{01} = -b_{10} = e\,, \tag{2.3}$$

with all other components of $b_{\mu\nu}$ being zero. Since a background b_{01} is equivalent to switching on an electric field on the D-brane world-volume, throughout this paper we shall refer to such background as background electric field. Eqs. (2.2) and (2.3) give:

$$G_{00} = -\left(1 - e^2\right), \qquad G_{11} = \left(1 - e^2\right), \qquad G_{ij} = \delta_{ij} \quad \text{for} \quad i, j \geq 2\,, \qquad \Theta^{01} = \frac{e}{(1 - e^2)}\,, \tag{2.4}$$

with all other components of $G_{\mu\nu}$ and $\Theta^{\mu\nu}$ being zero. $G_{\mu\nu}$ can be converted to $\eta_{\mu\nu}$ by a rescaling of x^0 and x^1 by $\sqrt{1-e^2}$. Thus if $T = F(x^0)$ is a solution of the equations of motion of $\mathcal{S}(T; \eta_{\mu\nu}, b_{\mu\nu} = 0)$, then $T = F(\sqrt{1-e^2}\, x^0)$ will be a solution of the equations of motion of $\mathcal{S}(T; G_{\mu\nu}, b_{\mu\nu} = 0)$ and also of $\mathcal{S}_\Theta(T; G_{\mu\nu}, b_{\mu\nu} = 0)$. By the equivalence (2.1), it is then also a solution of the equations of motion of $\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu})$.

Thus if we knew the solution $F(x^0)$, we could find the solution of the equations of motion of $\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu})$. In actual practice, however, we do not know the solution $F(x^0)$, but only know the energy momentum tensor associated with this solution [18, 19]. Let us denote this by $T^{(0)}_{\mu\nu}(x^0)$:

$$T^{(0)}_{\mu\nu}\left(x^0\right) = -2\, \frac{\delta \mathcal{S}(T = F(x^0); G_{\mu\nu}, b_{\mu\nu} = 0)}{\delta G^{\mu\nu}(x)}\bigg|_{G_{\mu\nu} = \eta_{\mu\nu}}\,. \tag{2.5}$$

(Throughout this paper we shall omit writing the dependence of various closed string sources on coordinates transverse to the D-brane, which is just a delta function of the transverse coordinates.) Then the energy momentum tensor

$$\widetilde{T}_{\mu\nu}\left(x^0\right) = -2(-\det G)^{-1/2} \frac{\delta \mathcal{S}\left(T = F\left(\sqrt{1-e^2}\, x^0\right); G_{\mu\nu}, b_{\mu\nu} = 0\right)}{\delta G^{\mu\nu}(x)}\,, \tag{2.6}$$

[5]Related technique has been used earlier in [39].

for $G_{\mu\nu}$ given in (2.4), is related to $T^{(0)}_{\mu\nu}(x^0)$ by the standard transformation laws under $(x^0, x^1) \to \sqrt{1-e^2}\,(x^0, x^1)$. This gives

$$\begin{aligned}
&\widetilde{T}_{ab}\left(x^0\right) = \left(1-e^2\right) T^{(0)}_{ab}\left(\sqrt{1-e^2}\,x^0\right), \qquad \widetilde{T}_{ai}\left(x^0\right) = \sqrt{1-e^2}\,T^{(0)}_{ai}\left(\sqrt{1-e^2}\,x^0\right), \\
&\widetilde{T}_{ij}\left(x^0\right) = T^{(0)}_{ij}\left(\sqrt{1-e^2}\,x^0\right), \qquad \text{for} \quad a,b = 0,1\,, \quad i,j \geq 2\,.
\end{aligned} \tag{2.7}$$

For the case at hand, we know the explicit form of $T^{(0)}_{\mu\nu}$ from the analysis of [18, 19]. For definiteness let us consider the case of bosonic string theory. In this case:

$$T^{(0)}_{00} = \frac{1}{2}\mathcal{T}_p\left(1+\cos\left(2\pi\widetilde{\lambda}\right)\right), \qquad T^{(0)}_{r0} = 0\,, \qquad T^{(0)}_{rs} = -\mathcal{T}_p f\left(x^0\right)\delta_{rs}\,, \qquad \text{for} \quad r,s \geq 1\,. \tag{2.8}$$

Here $\mathcal{T}_p$ is the tension of the D-p-brane, $\widetilde{\lambda}$ is a parameter labelling the total energy of the system, and

$$f(x^0) = \frac{1}{1+\sin(\pi\widetilde{\lambda})e^{x^0}} + \frac{1}{1+\sin(\pi\widetilde{\lambda})e^{-x^0}} - 1\,. \tag{2.9}$$

Using eqs. (2.7), (2.8) we get

$$\begin{aligned}
\widetilde{T}_{00} &= \left(1-e^2\right)\frac{1}{2}\mathcal{T}_p\left(1+\cos\left(2\pi\widetilde{\lambda}\right)\right), \qquad \widetilde{T}_{11} = -\left(1-e^2\right)\mathcal{T}_p f\left(\sqrt{1-e^2}\,x^0\right), \\
\widetilde{T}_{ij} &= -\mathcal{T}_p f\left(\sqrt{1-e^2}\,x^0\right)\delta_{ij}\,, \qquad \text{for} \quad i,j \geq 2\,.
\end{aligned} \tag{2.10}$$

The quantities of interest to us are the energy momentum tensor and the source for the antisymmetric tensor field computed from the action $\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu})$:

$$T^{\mu\nu}(x) = 2\frac{\delta\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu})}{\delta g_{\mu\nu}(x)}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}, \qquad S^{\mu\nu}(x) = 2\frac{\delta\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu})}{\delta b_{\mu\nu}(x)}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}. \tag{2.11}$$

Thus we have

$$\delta\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu}) = \frac{1}{2}\int d^{p+1}x\,\left[T^{\mu\nu}(x)\delta g_{\mu\nu}(x) + S^{\mu\nu}(x)\delta b_{\mu\nu}(x)\right]. \tag{2.12}$$

We shall now consider space-time independent variations $\delta g_{\mu\nu}$ and $\delta b_{\mu\nu}$. In this case, eq. (2.1) gives

$$\begin{aligned}
\delta\mathcal{S}(T; g_{\mu\nu}, b_{\mu\nu}) &= \left[\delta\left(\sqrt{\frac{\det(g+b)}{\det G}}\right)\mathcal{S}_\Theta(T; G_{\mu\nu}, b_{\mu\nu}=0) + \sqrt{\frac{\det(g+b)}{\det G}}\,\delta\mathcal{S}_\Theta(T; G_{\mu\nu}, b_{\mu\nu}=0)\right] \\
&= \left[\delta\left(\sqrt{\frac{\det(g+b)}{\det G}}\right)\mathcal{S}(T; G_{\mu\nu}, b_{\mu\nu}=0) + \sqrt{\frac{\det(g+b)}{\det G}}\,\delta\mathcal{S}(T; G_{\mu\nu}, b_{\mu\nu}=0)\right],
\end{aligned} \tag{2.13}$$

where in the last line we have made use of the fact that for T dependent on only the x^0 coordinate we can ignore the Θ dependence of the action. Eq. (2.6) gives:

$$\delta\mathcal{S}(T; G_{\mu\nu}, b_{\mu\nu}=0) = -\frac{1}{2}\int d^{p+1}x\,\widetilde{T}_{\mu\nu}(x)\delta G^{\mu\nu}\,, \tag{2.14}$$

JHEP11(2002)047

where, using (2.2) we have,

$$
\begin{aligned}
\delta G^{00} &= -\left(1-e^2\right)^{-2}\left(\delta g_{00}+2e\delta b_{01}-e^2\delta g_{11}\right),\\
\delta G^{11} &= -\left(1-e^2\right)^{-2}\left(\delta g_{11}-2e\delta b_{01}-e^2\delta g_{00}\right),\\
\delta G^{ij} &= -\delta g_{ij}\,, \qquad \text{for} \quad i,j\geq 2\,,\\
\delta\left(\sqrt{\frac{\det(g+b)}{\det G}}\right) &= \frac{1}{2}\left(1-e^2\right)^{-3/2}\left(e^2\delta g_{00}-e^2\delta g_{11}+2e\delta b_{01}\right).
\end{aligned}
\tag{2.15}
$$

In order to evaluate the right hand side of (2.13), we also need to compute $\mathcal{S}(T;G_{\mu\nu},b_{\mu\nu}=0)$ for the solution $T=F(\sqrt{1-e^2}x^0)$. This is done as follows. Let us consider a space-time independent variation δG_{22} of G_{22} with all other $\delta G_{\mu\nu}=0$. In this case, for the background of the form considered here, the only contribution to $\delta\mathcal{S}(T;G_{\mu\nu},b_{\mu\nu}=0)$ will come from the overall multiplicative factor of $\sqrt{-\det G}$ in the lagrangian density. Thus we have:

$$
\delta\mathcal{S}(T;G_{\mu\nu},b_{\mu\nu}=0)=\frac{1}{2}\delta G_{22}\mathcal{S}(T;G_{\mu\nu},b_{\mu\nu}=0)\,, \tag{2.16}
$$

since $G^{22}=1$. On other hand eq. (2.6) gives

$$
\delta\mathcal{S}(T;G_{\mu\nu},b_{\mu\nu}=0)=-\frac{1}{2}\int d^{p+1}x(-\det G)^{1/2}\delta G^{22}\widetilde{T}_{22}=\frac{1}{2}\int d^{p+1}x(-\det G)^{1/2}\delta G_{22}\widetilde{T}_{22}\,, \tag{2.17}
$$

since $\delta G_{22}=-\delta G^{22}$. Comparing (2.16) and (2.17) we get

$$
\mathcal{S}(T;G_{\mu\nu},b_{\mu\nu}=0)=\int d^{p+1}x(-\det G)^{1/2}\widetilde{T}_{22}\,. \tag{2.18}
$$

Eqs. (2.12)–(2.15), and (2.18) now give:

$$
\begin{aligned}
\delta\mathcal{S}(T;g_{\mu\nu},b_{\mu\nu}) \equiv{}& \frac{1}{2}\int d^{p+1}x[T^{\mu\nu}\delta g_{\mu\nu}+S^{\mu\nu}\delta b_{\mu\nu}]\\
={}& \frac{1}{2}\int d^{p+1}x\Big[\delta g_{00}\Big\{\left(1-e^2\right)^{-3/2}\widetilde{T}_{00}-e^2\left(1-e^2\right)^{-3/2}\widetilde{T}_{11}+e^2\left(1-e^2\right)^{-1/2}\widetilde{T}_{22}\Big\}+\\
&+\delta g_{11}\Big\{\left(1-e^2\right)^{-3/2}\widetilde{T}_{11}-e^2\left(1-e^2\right)^{-3/2}\widetilde{T}_{00}-e^2\left(1-e^2\right)^{-1/2}\widetilde{T}_{22}\Big\}+\\
&+2e\delta b_{01}\Big\{\left(1-e^2\right)^{-3/2}\widetilde{T}_{00}-\left(1-e^2\right)^{-3/2}\widetilde{T}_{11}+\left(1-e^2\right)^{-1/2}\widetilde{T}_{22}\Big\}+\\
&+\left(1-e^2\right)^{1/2}\widetilde{T}_{ij}\delta g_{ij}\Big]\,.
\end{aligned}
\tag{2.19}
$$

Since $\delta g_{\mu\nu}$ and $\delta b_{\mu\nu}$ are arbitrary constants, from (2.19) we can compute space-time integrals of $T_{\mu\nu}$ and $S_{\mu\nu}$. If we assume that the relation holds also for the integrands, then we get

$$
\begin{aligned}
T_{00} &= T^{00} = \left[\frac{1}{2}e^2\left(1-e^2\right)^{-1/2}\mathcal{T}_p\left(1+\cos\left(2\pi\widetilde{\lambda}\right)\right)\right]+\left\{\frac{1}{2}\left(1-e^2\right)^{1/2}\mathcal{T}_p\left(1+\cos\left(2\pi\widetilde{\lambda}\right)\right)\right\},\\
T_{11} &= T^{11} = -\left[\frac{1}{2}e^2(1-e^2)^{-1/2}\mathcal{T}_p\left(1+\cos\left(2\pi\widetilde{\lambda}\right)\right)\right]-\left\{\left(1-e^2\right)^{1/2}\mathcal{T}_p f\left(\sqrt{1-e^2}\,x^0\right)\right\},\\
T_{ij} &= T^{ij} = -\left\{\left(1-e^2\right)^{1/2}\mathcal{T}_p f\left(\sqrt{1-e^2}\,x^0\right)\delta_{ij}\right\},\\
S_{01} &= -S^{01} = -\left[\frac{1}{2}e\left(1-e^2\right)^{-1/2}\mathcal{T}_p\left(1+\cos\left(2\pi\widetilde{\lambda}\right)\right)\right].
\end{aligned}
\tag{2.20}
$$

Note that in defining S_{01} we have taken into account the fact that δS receives contribution from $\frac{1}{2}S^{01}\delta b_{01}$ as well as $\frac{1}{2}S^{10}\delta b_{10}$.

We now need to examine to what extent it is justified to equate the integrands in (2.19) to arrive at (2.20). First of all, note that the dependence of $T_{\mu\nu}$ and $S_{\mu\nu}$ on the spatial coordinates is trivial (independent of the tangential directions, and proportional to a delta function involving the transverse coordinates which is understood in (2.20)). Thus the only question is if we can equate the integrands under the x^0 integral. Since $T_{0\mu}$ and $S_{0\mu}$ are time independent due to their conservation laws, the answers for these quantities given in (2.20) are certainly valid. As for the other quantities, we note that the detailed time dependence of these quantities depends to a large extent on the precise off-shell definition of the metric since we need to compute the change in the action under a local variation of the metric and anti-symmetric tensor field. As a result, these quantities can be changed by changing the definition of the off-shell continuation of the metric e.g. by redefining the off-shell vertex operator by a conformal transformation. Thus the expressions given in (2.20) are equally good choices to any other expressions consistent with the integrated equation (2.19). Nevertheless, we shall see in later sections that the boundary state analysis, which comes with a precise convention for coupling of off-shell closed string states to a D-brane, leads to the same expressions for the time evolution of the various sources as given in (2.20).

$T_{\mu\nu}$ and $S_{\mu\nu}$ given in (2.20) can be interpreted as a sum of the contribution from a configuration of fundamental strings (shown in square bracket) and from rolling tachyon in the absence of electric field (shown in curly brackets). Note however that the evolution of the contribution from rolling tachyon slows down by a factor of $\sqrt{1-e^2}$ in the presence of fundamental string charge. Thus the two systems do not decouple. We can get pure fundamental string background without any contribution from rolling tachyon by taking the limit

$$e \to 1, \qquad \widetilde{\lambda} \to \frac{1}{2}, \qquad (1-e^2)^{-1/2}\left(1+\cos\left(2\pi\widetilde{\lambda}\right)\right) \quad \text{fixed}. \tag{2.21}$$

However, as shown in appendix A, the contribution to the boundary state from higher level closed string states blows up in this limit. The meaning of this divergence is not entirely clear to us.

These results can be generalised to the case of decay of unstable D-p-branes and brane-antibrane systems in superstring theories. In fact the final formula for $T_{\mu\nu}$ and $S_{\mu\nu}$ are given by the same expressions as (2.20), all that changes is the expression for $f(x^0)$. In this case we have [19]

$$f(x^0) = \frac{1}{1+\sin^2(\pi\widetilde{\lambda})e^{\sqrt{2}x^0}} + \frac{1}{1+\sin^2(\pi\widetilde{\lambda})e^{-\sqrt{2}x^0}} - 1\,. \tag{2.22}$$

Similarly we can generalise the results to the case where the tachyon begins rolling at the top of the potential with a non-zero velocity so that the total energy of the system is larger than the tension of the brane. This requires a replacement of $\cos(2\pi\widetilde{\lambda})$ by $\cosh(2\pi\widetilde{\lambda})$ and appropriate replacements for $f(x^0)$ as given in [19].

JHEP11(2002)047

3. Boundary conformal field theory analysis

It was shown in ref. [18] that in the absence of background electric (or b) field, the class of time dependent solutions describing rolling of a D-p-brane away from the maximum of the tachyon potential is given by perturbing the boundary conformal field theory (BCFT) describing the original D-brane by the operator

$$\widetilde{\lambda} \int dt \cosh(X^0(t)) , \tag{3.1}$$

where $\widetilde{\lambda}$ is a constant parametrising the initial value of the tachyon, and t denotes the coordinate labelling the boundary of the world-sheet. This perturbation was identified in ref. [18] from the spatially homogeneous solution to the linearised equation of motion for the tachyon in string field theory. Such a deformation gives rise to a new BCFT since $\cosh(X^0)$ is an exactly marginal operator, and hence generates a solution of the equations of motion of open string theory.

In this section we shall generalise this construction to D-branes in the presence of a background electric field given in eq. (2.3). As discussed in the previous section, the space independent solution for the tachyon in presence of this non-trivial background can simply be obtained by incorporating the coordinate transformation $(x^0, x^1) \to \sqrt{1-e^2}\,(x^0, x^1)$ into the solution in absence of any background field. This, in turn means that the rolling tachyon solution in the presence of background field configuration $b_{01} = e$, is generated by the following deformation,

$$\widetilde{\lambda} \int dt \cosh\left(\sqrt{1-e^2} X^0(t)\right) . \tag{3.2}$$

In the following we shall show that the deformation (3.2) is exactly marginal, and analyse the deformed BCFT. We shall also find the boundary state associated with this BCFT.

3.1 The boundary conformal field theory

In this subsection we shall demonstrate that (3.2) generates an exactly marginal deformation of the BCFT describing the D-brane in the background (2.3). In order to do so we shall show that this represents an exactly marginal deformation in the Wick rotated theory obtained by the replacement $X^0 \to -iX^0$. Under this rotation the closed string metric $g_{\mu\nu}$ and the antisymmetric tensor field $b_{\mu\nu}$ go to $\bar{g}_{\mu\nu}$ and $\bar{b}_{\mu\nu}$ respectively, given by,

$$\bar{g}_{\mu\nu} = \delta_{\mu\nu} , \qquad \bar{b}_{01} = -\bar{b}_{10} = -ie \equiv \bar{e} , \tag{3.3}$$

with other components of $\bar{b}_{\mu\nu}$ being zero. After performing Wick rotation on the world-sheet as well, one gets the following boundary condition:

$$\left(\bar{g}_{\mu\nu}\partial_n X^\nu + i\bar{b}_{\mu\nu}\partial_t X^\nu\right)\Big|_{\partial\Sigma} = 0 , \tag{3.4}$$

where ∂_n and ∂_t denote respectively the normal and tangential derivatives at the boundary $\partial\Sigma$. For simplicity we have restricted our analysis to the case of D-25-brane, but the

JHEP11(2002)047

generalisation to an arbitrary D-p-brane is straightforward. The boundary conformal field theory at hand is same as the one [8] corresponding to an imaginary B-field $\bar{b}_{\mu\nu}$ switched on along spatial Neumann directions. In the following we shall consider the case of a real $\bar{b}$-field, i.e. a real magnetic field $\bar{e}$ in the 0-1 plane, and present all the relevant formulæ in that context. Any given formula can be translated to the one corresponding to an electric field by the analytic continuation,

$$\bar{e} \to -ie\,, \tag{3.5}$$

together with an inverse Wick rotation $X^0 \to iX^0$.

The expressions for the effective euclidean open string metric $\bar{G}$ and the non-commutativity parameter $\bar{\Theta}$ are given by,

$$\bar{G}^{\mu\nu} = \left[\left(\bar{g}+\bar{b}\right)^{-1}\right]^{\mu\nu}_{S}\,, \qquad \bar{\Theta}^{\mu\nu} = \left[\left(\bar{g}+\bar{b}\right)^{-1}\right]^{\mu\nu}_{A}\,. \tag{3.6}$$

It will be convenient for our discussion to introduce the vielbeins of the open and closed string metrics and the corresponding local coordinates. These are defined by the following equations:

$$\begin{aligned} \bar{G}_{\mu\nu} &= V^a_{\ \mu} V^a_{\ \nu} = \left(V^T V\right)_{\mu\nu}\,, \qquad \bar{g}_{\mu\nu} = v^a_{\ \mu} v^a_{\ \nu} = \left(v^T v\right)_{\mu\nu}\,, \\ Z^a &= V^a_{\ \mu} X^\mu\,, \qquad Y^a = v^a_{\ \mu} X^\mu\,, \end{aligned} \tag{3.7}$$

where $a,\ \mu = 0, \cdots, 25$ and the matrix notation for various quantities has the obvious meaning. For the case under study, we have

$$\bar{g} = \mathbb{1}_{26}\,, \qquad \bar{b} = \begin{pmatrix} 0 & \bar{e} & 0 \\ -\bar{e} & 0 & 0 \\ 0 & 0 & \mathbb{0}_{24} \end{pmatrix}, \tag{3.8}$$

$$\bar{G} = \begin{pmatrix} 1+\bar{e}^2 & 0 & 0 \\ 0 & 1+\bar{e}^2 & 0 \\ 0 & 0 & \mathbb{1}_{24} \end{pmatrix}, \qquad \bar{\Theta} = \begin{pmatrix} 0 & -\frac{\bar{e}}{1+\bar{e}^2} & 0 \\ \frac{\bar{e}}{1+\bar{e}^2} & 0 & 0 \\ 0 & 0 & \mathbb{0}_{24} \end{pmatrix}, \tag{3.9}$$

and we shall choose

$$V = \begin{pmatrix} \sqrt{1+\bar{e}^2} & 0 & 0 \\ 0 & \sqrt{1+\bar{e}^2} & 0 \\ 0 & 0 & \mathbb{1}_{24} \end{pmatrix}, \qquad v = \begin{pmatrix} \frac{1}{\sqrt{1+\bar{e}^2}} & \frac{\bar{e}}{\sqrt{1+\bar{e}^2}} & 0 \\ -\frac{\bar{e}}{\sqrt{1+\bar{e}^2}} & \frac{1}{\sqrt{1+\bar{e}^2}} & 0 \\ 0 & 0 & \mathbb{1}_{24} \end{pmatrix}. \tag{3.10}$$

The relation between the two local frames is given by,

$$Z = LY\,, \qquad L = Vv^{-1} = \begin{pmatrix} 1 & -\bar{e} & 0 \\ \bar{e} & 1 & 0 \\ 0 & 0 & \mathbb{1}_{24} \end{pmatrix}. \tag{3.11}$$

The boundary two point functions [1, 2, 8] of the local coordinates Z^a take the form ($\alpha' = 1$),

$$\langle Z^a(x) Z^b(y)\rangle = -\delta^{ab}\log(x-y)^2 + i\ \pi\ \bar{\Theta}^{ab}_Z \epsilon(x-y)\,, \tag{3.12}$$

where,

$$\bar{\Theta}_Z = V\bar{\Theta}V^T\,, \tag{3.13}$$

and $\epsilon(x)$ is the sign function. Let us now define the following boundary operators,

$$J^1_{Z^a}(x) = \cos\left(Z^a(x)\right), \qquad J^2_{Z^a}(x) = \sin\left(Z^a(x)\right), \qquad J^3_{Z^a}(x) = \frac{i}{2}\partial Z^a(x). \tag{3.14}$$

Using the two point function (3.12) it is straightforward to show that these operators have conformal dimension 1. We shall now argue along the line of ref. [32] that the operator

$$\int dt\, \mathcal{O}^{Z^0}_{\widetilde{\lambda}}(t) \equiv \widetilde{\lambda} \int dt\, J^1_{Z^0}(t) = \widetilde{\lambda} \int dx\, \cos\left(\sqrt{1+\bar{e}^2}\, X^0(t)\right), \tag{3.15}$$

describes an exactly marginal deformation. It was shown in [32] that (3.15) describes an exactly marginal deformation if $\mathcal{O}^{Z^0}_{\widetilde{\lambda}}$ is *self-local*, *i.e.* in a correlation function involving $\mathcal{O}^{Z^0}_{\widetilde{\lambda}}(x)\mathcal{O}^{Z^0}_{\widetilde{\lambda}}(y)$, the result for $y > x$ is related to that for $y < x$ by an analytic continuation in the complex y plane, and the result of the analytic continuation does not depend on whether it is done in the upper half plane or the lower half plane. Using the two point function (3.12) it is straightforward to verify that this condition is satisfied.[6]

This establishes the exact marginality of the operator $\cos(\sqrt{1+\bar{e}^2}X^0)$. Making the analytic continuation $\bar{e} \to -ie$ and the inverse Wick-rotation $X^0 \to iX^0$ we arrive at the original BCFT with a time-like direction where the above marginal operator becomes $\cosh(\sqrt{1-e^2}X^0)$. Thus we can perturb the original theory by $\widetilde{\lambda}\int dt \cosh(\sqrt{1-e^2}X^0(t))$ to generate rolling tachyon solutions in the presence of electric field.

In order to find the energy-momentum tensor associated with this rolling tachyon solution, we need to evaluate the boundary state associated with the deformed BCFT. This can be done by first working with the euclidean theory with real magnetic field $\bar{e}$ and then making the replacement $X^0 \to iX^0$, $\bar{e} \to -ie$. We shall do this next and show that the result agrees with the ones found in section 2. The construction in the euclidean theory will proceed according to the following steps:

1. First we shall compactify the theory in a specific manner, and construct the boundary state $|\mathcal{B}_{\bar{b}}\rangle$ corresponding to the unperturbed BCFT ($\widetilde{\lambda} = 0$) including the background magnetic field with this specific compactification.

2. Then we study the effect of switching on the perturbation $\widetilde{\lambda}$ on the boundary state in the compactified theory, and compute the deformed boundary state $|\mathcal{B}_{\bar{b},\widetilde{\lambda}}\rangle$.

3. Finally we take the decompactification limit by removing all the winding modes in the expression for the boundary state. This gives the deformed boundary state $|\mathcal{B}^{\infty}_{\bar{b},\widetilde{\lambda}}\rangle$ in the non-compact theory.

[6] This argument can be easily extended to the case of the arbitrary linear combination $\mathcal{O}^{Z^a}_{\widetilde{\lambda}_1,\widetilde{\lambda}_2} = \widetilde{\lambda}_1 J^1_{Z^a} + \widetilde{\lambda}_2 J^2_{Z^a}$. In fact using the notion of *mutual locality* between two operators introduced in [32] one can show that there exist special values of the magnetic field, namely $\bar{e} = n, n \in \mathbb{Z}$ for which $\mathcal{O}^{Z^a}_{\widetilde{\lambda}_1,\widetilde{\lambda}_2}$ and $\mathcal{O}^{Z^b}_{\widetilde{\lambda}_1,\widetilde{\lambda}_2}$, for $a \neq b$, become mutually local, and hence any linear combination of them represents a marginal deformation. These correspond to the special conformal points (magic) found in [30] where two cosine perturbations were simultaneously switched on along orthogonal directions in presence of a magnetic field. These special conformal points do not occur in the electric theory due to the fact that the non-commutativity parameter is imaginary in this case.

3.2 Compactification and enhancement of SU(2) symmetries

Following ref. [31, 32] we shall first construct the boundary state in a theory with suitable compactification and then go to its universal cover to get the answer in the non-compact limit. In this subsection, therefore we shall discuss some issues involving compactifications. We want to construct the boundary state corresponding to the marginal deformation given by the operator (3.15). Therefore the compactifications of our interest are the ones that maintain the periodicity of this operator. At the same time, we would like to ensure that in the compactified theory there is an enhanced $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry in the closed string sector which can be used to organise the boundary state as in [31, 32]. Naively one might think that periodicity of $\cos(Z^0)$ would require compactifying Z^0 on a circle of unit radius keeping Z^1 non-compact, i.e. making the identification $(Z^0, Z^1) \equiv (Z^0 + 2\pi, Z^1)$. However in this case the radius of the compact circle, measured in the closed string metric, is $1/\sqrt{1+\bar{e}^2}$, and hence we do not get enhanced $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gauge symmetry in the closed string sector. The compactification which achieves this is

$$\left(Y^0, Y^1\right) \equiv \left(Y^0 + 2\pi, Y^1\right). \tag{3.16}$$

From (3.11) we have

$$\begin{aligned}
& Z^0 = Y^0 - \bar{e} Y^1, \qquad Z^1 = \bar{e} Y^0 + Y^1, \qquad Z^\mu = Y^\mu \quad \text{for} \quad \mu \geq 2 \\
\rightarrow \qquad & Y^0 = \frac{Z^0}{1+\bar{e}^2} + \frac{\bar{e} Z^1}{1+\bar{e}^2}, \qquad Y^1 = -\frac{\bar{e} Z^0}{1+\bar{e}^2} + \frac{Z^1}{1+\bar{e}^2}.
\end{aligned} \tag{3.17}$$

This, together with (3.16) gives

$$\left(Z^0, Z^1\right) \equiv \left(Z^0 + 2\pi, Z^1 + 2\pi\bar{e}\right). \tag{3.18}$$

The operator (3.15) is clearly invariant under this transformation. On the other hand since in the Y^μ coordinate system the closed string metric is identity, compactifying Y^0 with period 2π implies that the radius of the circle, measured in closed string metric, is 1. Thus the closed string theory now has enhanced $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gauge symmetry which can be used to organise the boundary state as in [31, 32]. The left moving currents are given by,

$$J^1_{Y^0_L}(u) = \cos\left(2Y^0_L(u)\right), \qquad J^2_{Y^0_L}(u) = \sin\left(2Y^0_L(u)\right), \qquad J^3_{Y^0_L}(u) = i\partial Y^0_L(u). \tag{3.19}$$

Note that all the $J^i_{Y^0_L}$ are well defined operators since Y^0 is compactified on a circle of self-dual radius.

3.3 The boundary state in the compactified theory

We shall now construct the boundary state $|\mathcal{B}_{\bar{b}}\rangle$ for an euclidean D25-brane with a magnetic $\bar{b}$-field turned on, and Y^0 compactified on a circle of unit radius. It can be expressed as,

$$|\mathcal{B}_{\bar{b}}\rangle = |\mathrm{Bos}; \bar{b}\rangle \otimes |\mathrm{ghost}\rangle, \tag{3.20}$$

where $|\mathrm{Bos};\bar{b}\rangle$ and $|\mathrm{ghost}\rangle$ represent the bosonic and ghost parts of the boundary state respectively. We have

$$|\mathrm{ghost}\rangle|\rangle = \exp\left(-\sum_{n\geq 0}\left(\bar{b}_{-n}c_{-n} + b_{-n}\bar{c}_{-n}\right)\right)(c_0 \ + \ \bar{c}_0)c_1\bar{c}_1|0\rangle\,. \tag{3.21}$$

where the ghost oscillators have their usual definition. Construction of the bosonic part follows [2, 30]. The closed string overlap condition on the cylinder corresponding to the open string boundary condition (3.4) reads,

$$\left(\bar{g}\,\partial_\tau X + i\bar{b}\,\partial_\sigma X\right)\Big|_{\tau=0} = 0\,. \tag{3.22}$$

This, in turn, gives rise to the following condition on $|\mathcal{B}_{\bar{b}}\rangle$ in terms of the oscillators σ_n's and $\bar{\sigma}_n$'s of the coordinates Y,

$$[\sigma_n + M_Y\ \bar{\sigma}_{-n}]\,|\mathcal{B}_{\bar{b}}\rangle = 0\,, \qquad \forall n\in\mathbb{Z}\,, \tag{3.23}$$

where the matrix M_Y is given by,

$$M_Y = \begin{pmatrix} \frac{1-\bar{e}^2}{1+\bar{e}^2} & \frac{-2\bar{e}}{1+\bar{e}^2} & 0 \\ \frac{2\bar{e}}{1+\bar{e}^2} & \frac{1-\bar{e}^2}{1+\bar{e}^2} & 0 \\ 0 & 0 & 1\!\!1_{24} \end{pmatrix}. \tag{3.24}$$

Using (3.23) we get,

$$|\mathrm{Bos};\bar{b}\rangle = N_{\bar{b}}\exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\ \sigma_{-n}^T\ M_Y\ \bar{\sigma}_{-n}\right]|\mathrm{Bos};\bar{b}\rangle_0\,, \tag{3.25}$$

where $|\mathrm{Bos};\bar{b}\rangle_0$ is the zero mode part of the boundary state. Since we are considering a D25-brane, the normalisation constant is given by [36],

$$N_{\bar{b}} = K\mathcal{T}_{25}\sqrt{\det\left(\bar{g}+\bar{b}\right)}\,. \tag{3.26}$$

Here $\mathcal{T}_{25}$ being the D25-brane tension and K is a convention dependent numerical factor independent of $\bar{b}$. Our final answer for the energy momentum tensor will be independent of the choice of K.

To construct the zero mode part $|\mathrm{Bos};\bar{b}\rangle_0$ let us first also compactify the coordinate Y^1 on a circle of radius R. We shall finally take the $R\to\infty$ limit to get the desired result. Since all the coordinates except for Y^0 and Y^1 are noncompact and satisfy Neumann boundary condition, $|\mathrm{Bos};\bar{b}\rangle_0$ contains only the momentum and winding mode excitations coming from Y^0 and Y^1. If $n^{(a)}, w^{(a)} \in \mathbb{Z}, a = 0,1$ are respectively momentum and winding numbers then the above overlap condition for the zero-modes restricts the eigenvalues appearing in $|\mathrm{Bos};\bar{b}\rangle_0$ in the following way,

$$\left(\frac{n^{(a)}}{R^{(a)}} + w^{(a)}R^{(a)}\right) + M_Y{}^a{}_b\left(\frac{n^{(b)}}{R^{(b)}} - w^{(b)}R^{(b)}\right) = 0\,. \tag{3.27}$$

Using the explicit matrix value for M_Y one reduces the above equation to:

$$w^{(0)}R^{(0)} = \frac{n^{(1)}}{\bar{e}R^{(1)}}, \qquad w^{(1)}R^{(1)} = -\frac{n^{(0)}}{\bar{e}R^{(0)}}, \qquad R^{(0)} = 1, \qquad R^{(1)} = R\,. \tag{3.28}$$

So we see that only two of the four variables $n^{(a)}$, $w^{(a)}$ are independent. At a given finite value of R the restriction on $\bar{e}$ is given by the flux quantisation law: $\bar{e}R = a$, where a is an integer. This gives $n^{(1)} = aw^{(0)}$, $n^{(0)} = aw^{(1)}$. Thus the sum can be taken over integer values of $w^{(0)}$, $w^{(1)}$ and we have,

$$\begin{aligned}|\mathrm{Bos};\bar{b}\rangle_0 \;=\; \sum_{w^{(0)},w^{(1)}\in\mathbb{Z}} \exp\Big[&i\left(-\bar{e}w^{(1)}R + w^{(0)}\right)y_L^0 + i\left(w^{(1)}R + \bar{e}w^{(0)}\right)y_L^1 + \\ &+ i\left(-\bar{e}w^{(1)}R - w^{(0)}\right)y_R^0 + i\left(-w^{(1)}R + \bar{e}w^{(0)}\right)y_R^1\Big]|0\rangle\,. \end{aligned} \tag{3.29}$$

The non-compact limit, $R \to \infty$ is obtained by keeping only the $w^{(1)} = 0$ term in the sum over $w^{(1)}$. This gives

$$|\mathrm{Bos};\bar{b}\rangle_0 = \sum_{m\in\mathbb{Z}/2} \exp\left[-2im\left(y_L^0 - y_R^0\right) - 2im\bar{e}\left(y_L^1 + y_R^1\right)\right]|0\rangle\,. \tag{3.30}$$

Note that in this limit $\bar{e}$ can take any real value.

We now note from eq. (3.24) that M_Y is an orthogonal matrix of determinant one. Then the oscillators,

$$\bar{\tau}_n = M_Y\bar{\sigma}_n = v^{-1}\bar{\alpha}_n\,, \qquad n \neq 0\,, \tag{3.31}$$

where α_n^μ, $\bar{\alpha}_n^\mu$ denote the oscillators of X^μ, obey the same commutation relations as $\bar{\sigma}_n$'s and therefore as far as construction of basis states is concerned they are as good as the $\bar{\sigma}_n$ oscillators. Moreover all the Virasoro oscillators $\bar{L}_n$'s take the same form in terms of the $\bar{\tau}$ oscillators as in terms of the $\bar{\sigma}$ oscillators. Using these facts one can write the state $|\mathrm{Bos};\bar{b}\rangle$ given in (3.25), (3.30) in the following form,

$$\begin{aligned}|\mathrm{Bos};\bar{b}\rangle \;&=\; N_{\bar{b}}\exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\left(\sigma_{-n}^0\bar{\tau}_{-n}^0 + \sigma_{-n}^1\bar{\tau}_{-n}^1 + \sum_{i=2}^{25}\alpha_{-n}^i\bar{\alpha}_{-n}^i\right)\right]\times \\ &\quad\times\sum_{m\in\mathbb{Z}/2}\exp\left[-2im\left(y_L^0 - y_R^0\right) - 2im\bar{e}y^1\right]|0\rangle \\ &= N_{\bar{b}}\sum_{j,m}|j,-m,m\rangle\rangle_{\bar{\tau}}^{(0)}\otimes\exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\sigma_{-n}^1\bar{\tau}_{-n}^1 - 2im\bar{e}y^1\right]|0\rangle\otimes|N\rangle_{c=24}\,, \end{aligned} \tag{3.32}$$

where

$$|N\rangle_{c=24} = \exp\left[-\sum_{n\geq1,i\geq2}\frac{1}{n}\alpha_{-n}^i\bar{\alpha}_{-n}^i\right]|0\rangle\,. \tag{3.33}$$

The state $|j,-m,m\rangle\rangle_{\bar{\tau}}^{(0)}$ is constructed by following the two steps: construct the Virasoro Ishibashi state $|j,-m,m\rangle\rangle^{(0)}$ in the representation of $\mathrm{SU}(2)_{Y_L^0}\otimes\mathrm{SU}(2)_{Y_R^0}$ [31, 32]. Then replace the $\bar{\sigma}$ oscillators by the corresponding $\bar{\tau}$ oscillators on the right part of the states appearing in the expansion of $|j,-m,m\rangle\rangle^{(0)}$.

3.4 Boundary state in the deformed theory

We shall now turn to the boundary state corresponding to the deformation of the conformal field theory by the boundary operator,

$$\int dt\, \mathcal{O}^{Z^0}_{\widetilde{\lambda}}(t) = \widetilde{\lambda} \int dt\, \cos\left(Z^0(t)\right) . \tag{3.34}$$

Using the boundary condition (3.23) we can show that on the boundary $Z^0(t) = 2Y^0_L(t)$. This allows us to replace $\cos(Z^0(t))$ in (3.34) by $\cos(2Y^0_L(t)) = J^1_{Y^0_L}(t)$. Now, if $|\mathcal{B}_{\bar{b},\widetilde{\lambda}}\rangle$ denotes the boundary state in the presence of this perturbation, then, given any closed string vertex operator ϕ_c of ghost number 3, the one point function of ϕ_c on a unit disk in the perturbed BCFT is given by $\langle\mathcal{B}_{\bar{b},\widetilde{\lambda}}|\phi_c\rangle$. Thus we have:[7]

$$\langle\mathcal{B}_{\bar{b},\widetilde{\lambda}}|\phi_c\rangle = \langle\mathcal{B}_{\bar{b}}|\exp\left[2\pi i\widetilde{\lambda}\, Q^1_{Y^0_L}\right]|\phi_c\rangle\,, \tag{3.35}$$

where the operators $Q^i_{Y^0_L}$'s are the $\mathrm{SU}(2)_L$ charges in the closed string theory given by,

$$Q^i_{Y^0_L} = \oint \frac{du}{2\pi i}\, J^i_{Y^0_L}(u)\,. \tag{3.36}$$

This gives:

$$\begin{aligned}
|\mathcal{B}_{\bar{b},\widetilde{\lambda}}\rangle &= \exp\left[-2\pi i\widetilde{\lambda}\, Q^1_{Y^0_L}\right]|\mathcal{B}_{\bar{b}}\rangle \\
&= N_{\bar{b}} \sum_{j,m} \exp\left[-2\pi i\tilde{\lambda} Q^1_{Y^0_L}\right]|j,-m,m\rangle\rangle^{(0)}_{\bar{\tau}} \otimes \exp\left[-\sum_{n>0}\frac{1}{n}\sigma^1_{-n}\bar{\tau}^1_{-n} - 2im\bar{e}y^1\right]|0\rangle \otimes \\
&\quad \otimes|N\rangle_{c=24}\otimes|\mathrm{ghost}\rangle, \\
&= N_{\bar{b}} \sum_{j,m,m'} D^j_{m',-m}|j,m',m\rangle\rangle^{(0)}_{\bar{\tau}} \otimes \exp\left[-\sum_{n>0}\frac{1}{n}\sigma^1_{-n}\bar{\tau}^1_{-n} - 2im\bar{e}y^1\right]|0\rangle \otimes \\
&\quad \otimes|N\rangle_{c=24}\otimes|\mathrm{ghost}\rangle\,,
\end{aligned} \tag{3.37}$$

where $D^j_{m',m}$ is simply the spin j representation matrix of the operator $\exp(-2\pi i\widetilde{\lambda}Q^1_{Y^0_L})$. The values of these matrix elements can be obtained by using the formula given in ref. [32] with the $j = \frac{1}{2}$ representation matrix given in ref. [18]. Notice that the left parts of all the states appearing in the expansion of $|j,m',m\rangle\rangle^{(0)}_{\bar{\tau}}$ transform in the same and well defined way under $\mathrm{SU}(2)_{Y^0_L}$ while the right parts of the states do not obey simple transformation rules under $\mathrm{SU}(2)_{Y^0_R}$. But this is sufficient for us to be able to compute the action of the rotation operator $\exp\left[-2\pi i\tilde{\lambda}Q^1_{Y^0_L}\right]$.

[7] It is possible to reduce the integrated boundary deformation in the action to an exponentiated operator acting on the Hilbert space if the deformation is exactly marginal. This was shown in [32] very simply by using the self-locality property. Without using self-locality one can also show this by following the systematic procedure of renormalisation as was done in hep-th/9402113 in [31].

JHEP11(2002)047

Finally we can take the non-compact limit of the above boundary state by simply removing all the winding sector states from the boundary state (3.37). In this limit, of the $|j,m',m\rangle\rangle$ only the $|j,m,m\rangle\rangle$ states survive [31, 32, 30]. Thus we get the final result,

$$
\begin{aligned}
|\mathcal{B}^{\infty}_{\bar{b},\tilde{\lambda}}\rangle &= N_{\bar{b}} \sum_{j,m} D^{j}_{m,-m} |j,m,m\rangle\rangle^{(0)}_{\bar{\tau}} \otimes \exp\left[-\sum_{n>0} \frac{1}{n} \sigma^{1}_{-n} \bar{\tau}^{1}_{-n} - 2im\bar{e}y^{1} \right] |0\rangle \otimes \\
&\quad \otimes |N\rangle_{c=24} \otimes |\text{ghost}\rangle .
\end{aligned} \tag{3.38}
$$

3.5 Sources from boundary state

In this subsection we shall use the boundary state (3.38) to compute the level one sources, namely the energy-momentum tensor $T_{\mu\nu}$ and the source $S_{\mu\nu}$ for the antisymmetric tensor B-field. Since we are interested in getting the result for the theory with electric field and the boundary deformation (3.2), we need to perform a two-fold operation of inverse Wick-rotation $X^0 \to iX^0$ and the analytic continuation (3.5) on the quantities we get directly from the boundary state (3.38).

Following ref. [19] we first notice that the level (1,1) part of $|\mathcal{B}^{\infty}_{\bar{b},\tilde{\lambda}}\rangle$, after the two-fold operation, has the following general form:

$$
\int d^{26}k \left[\left(\tilde{A}_{\mu\nu}(k) + \tilde{C}_{\mu\nu}(k) \right) \alpha^{\mu}_{-1} \bar{\alpha}^{\nu}_{-1} + \tilde{B}(k) \left(b_{-1}\bar{c}_{-1} + \bar{b}_{-1}c_{-1} \right) \right] (c_0 + \bar{c}_0) c_1 \bar{c}_1 |k\rangle , \tag{3.39}
$$

where $\tilde{A}_{\mu\nu} = \tilde{A}_{\nu\mu}$ and $C_{\mu\nu} = -\tilde{C}_{\nu\mu}$. The conservation law $(Q_B + \bar{Q}_B)|\mathcal{B}_{\bar{b},\tilde{\lambda}}\rangle = 0$ obtained from this part of the boundary state reads in the coordinate space,

$$
\partial^{\nu}(A_{\mu\nu}(x) + \eta_{\mu\nu}B(x)) = 0 , \qquad \partial^{\nu} C_{\mu\nu}(x) = 0 , \tag{3.40}
$$

where $A_{\mu\nu}$, $C_{\mu\nu}$ and B are the Fourier transforms of $\tilde{A}_{\mu\nu}$, $\tilde{C}_{\mu\nu}$ and $\tilde{B}$ respectively. This gives the following two conserved currents,

$$
T_{\mu\nu}(x) = K_s(A_{\mu\nu}(x) + \eta_{\mu\nu}B(x)) , \qquad S_{\mu\nu}(x) = K_a C_{\mu\nu}(x) , \tag{3.41}
$$

where K_s and K_a are two appropriate normalisation constants.

We shall now compute $A_{\mu\nu}(x)$, $C_{\mu\nu}(x)$ and $B(x)$ from $|\mathcal{B}^{\infty}_{\bar{b},\tilde{\lambda}}\rangle$. The non-trivial part of this calculation is the contribution from the Y^0, Y^1 parts up to level (1,1). This can be obtained by replacing the α^0_{-1}, $\bar{\alpha}^0_{-1}$, α^1_{-1}, $\bar{\alpha}^1_{-1}$ oscillators in the result of [18, 19] by $-i\sigma^0_{-1}$, $-i\bar{\tau}^0_{-1}$, σ^1_{-1}, $\bar{\tau}^1_{-1}$ respectively, and X^0 by $-iZ^0 = -i(Y^0 - \bar{e}Y^1)$.[8] The result is proportional to:

$$
\begin{aligned}
&\left[\left(1 - \sigma^1_{-1}\bar{\tau}^1_{-1}\right) \hat{f}\left(Y^0(0) - \bar{e}Y^1(0)\right) - \sigma^0_{-1}\bar{\tau}^0_{-1}\, \hat{g}\left(Y^0(0) - \bar{e}Y^1(0)\right) \right] |0\rangle_{Y^0,Y^1} = \\
&\quad = \left[\left(1 - \sigma^1_{-1}\bar{\tau}^1_{-1}\right) \hat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right) - \sigma^0_{-1}\bar{\tau}^0_{-1}\, \hat{g}\left(\sqrt{1+\bar{e}^2}X^0(0)\right) \right] |0\rangle_{Y^0,Y^1} ,
\end{aligned} \tag{3.42}
$$

[8] The factors of $-i$ reflect that we are still in the euclidean theory, whereas the results of [18, 19] were given after inverse Wick rotation.

where

$$\widehat{f}(x) = \left[1 + \sum_{n=1}^{\infty}\left(-\sin\left(\tilde{\lambda}\pi\right)\right)^n \left(e^{inx} + e^{-inx}\right)\right] = f(-ix)\,,$$
$$\widehat{g}(x) = 1 + \cos\left(2\pi\tilde{\lambda}\right) - \widehat{f}(x)\,, \tag{3.43}$$

with $f(x)$ defined as in eq. (2.9).

We can now determine B, $A_{\mu\nu}$ and $C_{\mu\nu}$ by comparing (3.38) with (3.39), and using (3.42). We also need to use (3.31) and the fourth equation in (3.7) along with the explicit matrix values of M_Y and v given in eqs. (3.24) and (3.10) respectively, to translate the σ^a_{-1} and $\bar{\tau}^a_{-1}$ into the oscillators α^μ_{-1} and $\bar{\alpha}^\mu_{-1}$ of X^μ. After the replacement $x^0 \to ix^0$, $\bar{e} \to -ie$, one gets,

$$\begin{aligned}
B(x) &= -K\mathcal{T}_{25}\sqrt{1-e^2}\,f\left(\sqrt{1-e^2}x^0\right)\,,\\
A_{00}(x) &= K\mathcal{T}_{25}\left[(1-e^2)^{-1/2}(1+\cos(2\pi\tilde{\lambda})) - (1-e^2)^{1/2}f(\sqrt{1-e^2}x^0)\right],\\
A_{11}(x) &= -K\mathcal{T}_{25}\left[e^2(1-e^2)^{-1/2}(1+\cos(2\pi\tilde{\lambda})) + (1-e^2)^{1/2}f(\sqrt{1-e^2}x^0)\right],\\
A_{ij} &= -K\mathcal{T}_{25}\,\delta_{ij}\left[(1-e^2)^{1/2}f(\sqrt{1-e^2}x^0)\right], \quad i,j\geq 2,\\
C_{01} &= K\mathcal{T}_{25}\left[e(1-e^2)^{-1/2}(1+\cos(2\pi\tilde{\lambda}))\right]\,,
\end{aligned} \tag{3.44}$$

with all other components of $A_{\mu\nu}$ and $C_{\mu\nu}$ being zero.

Using eqs. (3.41), (3.44) and (3.44) we get the following non-trivial components for the sources,

$$\begin{aligned}
T_{00} &= K_s K\mathcal{T}_{25}\left(1-e^2\right)^{-1/2}\left(1+\cos\left(2\pi\tilde{\lambda}\right)\right),\\
T_{11} &= -K_s K\mathcal{T}_{25}\left[e^2\left(1-e^2\right)^{-1/2}\left(1+\cos b\left(2\pi\tilde{\lambda}\right)\right) + 2\left(1-e^2\right)^{1/2}f\left(\sqrt{1-e^2}\,x^0\right)\right],\\
T_{ij} &= -2K_s K\mathcal{T}_{25}\left(1-e^2\right)^{1/2}f\left(\sqrt{1-e^2}\,x^0\right)\delta_{ij},\\
S_{01} &= K_a K\mathcal{T}_{25}e\left(1-e^2\right)^{-1/2}\left(1+\cos\left(2\pi\tilde{\lambda}\right)\right)\,.
\end{aligned} \tag{3.45}$$

Before comparing these results with the ones obtained from the target space analysis in section 2 (eqs. (2.20)) we shall determine the constants K_s and K_a from the known results for the boundary state and the Dirac-Born-Infeld action at $\tilde{\lambda} = 0$. The results for the sources obtained from this boundary state can simply be derived by taking the $\tilde{\lambda} \to 0$ limit of the results (3.45). Now the corresponding world-volume action takes the following standard form,

$$\mathcal{S}_{DBI} = -\mathcal{T}_p \int d^{p+1}x\sqrt{-\det(g+b)}. \tag{3.46}$$

It is straightforward to compute the sources from this action. For example, one gets the following results for T_{00} and S_{01},[9]

$$T_{00} = \mathcal{T}_p\,(1-e^2)^{-1/2}\,, \qquad S_{01} = -\mathcal{T}_p\,e(1-e^2)^{-1/2}\,. \tag{3.47}$$

[9]As in section 2, here also S^{01} has been defined in such a way that $\delta\mathcal{S}_{DBI}$ receives contribution both from $\frac{1}{2}S^{01}\delta b_{01}$ and $\frac{1}{2}S^{10}\delta b_{10}$.

Comparing these with the corresponding results in (3.45) in the limit $\tilde{\lambda} \to 0$ then fixes the constants to be,

$$K_s = \frac{1}{2K}\,, \qquad K_a = -\frac{1}{2K}\,. \tag{3.48}$$

With these values of the constants the results (3.45) exactly match with (2.20).

Given the ambiguity mentioned in the paragraph below (2.20), one might wonder why the boundary state analysis leads precisely to the same expressions as (2.20). In the boundary state formalism the coupling of off-shell closed strings is defined by inserting the corresponding closed string vertex operator at the center of the disk. To see why this prescription leads to (2.20), note that if (2.15) had held for arbitrary time dependent $\delta g_{\mu\nu}$ and $\delta b_{\mu\nu}$, then we could derive (2.20) explicitly using the target space analysis of section 2. This would require that for the relevant computations, the Seiberg-Witten equivalence relation holds even when the background metric and anti-symmetric tensor fields differ from constant values by an infinitesimal amount which depends on the space-time coordinates. Since in the boundary state analysis the x^0 dependence of various source terms (in the euclidean theory) is computed using an insertion of a bulk operator $e^{ik.X^0(0)}$ at the center of the disk, the agreement of boundary state analysis with (2.20) can be explained if in the computation of correlation functions of a single $e^{ik.X^0(0)}$ at the center, and open string vertex operators $\prod_i e^{ik_i.X^0(z_i)}$ inserted at the boundary, we can continue to use the open string metric even though there is a bulk operator insertion. This is indeed the case as long as there is only one insertion of the bulk operator $e^{ik.X^0}$, since such a correlation function does not involve the bulk propagator, and the bulk-boundary propagator, like the boundary propagator, depends only on the open string metric [8].[10]

4. Generalisation to superstrings

The boundary CFT analysis can be easily generalised to decay of non-BPS D-branes in superstring theory[11] in the presence of an electric field e. Here we shall extract the sources for massless closed string fields from the boundary state, as was done in [19], and not attempt to construct the full boundary state.[12] Since the analysis is a straightforward extension of the results of the previous section and [19], we shall only give the outlines of the derivation.

We work in the Wick rotated euclidean theory as in the case of bosonic string theory, and define the various coordinates Y^a, Z^a etc. in an identical manner. We also need to define the fermionic partners of various coordinates, and in general we shall denote the fermionic partners of X^μ, Y^a and Z^a by ψ_x^μ, ψ_y^a and ψ_z^a respectively. The boundary perturbation describing the Wick rotated rolling tachyon background is given by,

$$\widetilde{\lambda} \int dt\, \psi_z^0 \sin\left(Z^0/\sqrt{2}\right) \otimes \sigma_1\,, \tag{4.1}$$

[10] In any case, as long as the operator $e^{ik.X^0}$ is inserted at the center of the disk, the distance from the center of the disk to every point on the boundary is unity, and hence the correlator does not even involve the bulk-boundary propagator as it appears in the exponent of unity.

[11] See [40] and references therein.

[12] See [33] for discussions on SU(2) boundary states in the context of superstrings.

where σ_1 is a Chan-Paton factor. Using manipulations similar to those in bosonic string theory one can show that this can be rewritten as

$$\widetilde{\lambda} \int dt\, \psi^0_{yL} \sin\left(\sqrt{2} Y^0_L\right) \otimes \sigma_1 \,. \tag{4.2}$$

We can now construct the boundary state by first taking Y^0 to be a compact coordinate with radius $\sqrt{2}$, and then recovering the result for the non-compact case by throwing away all the winding modes. For compact Y^0 coordinate we can represent the bosonic coordinate Y^0 by a pair of fermionic coordinates (ξ, η) satisfying the following relations:

$$\begin{aligned} e^{i\sqrt{2}Y^0_L} &= \frac{1}{\sqrt{2}} \left(\xi_L(t) + i\eta_L(t)\right), \\ e^{i\sqrt{2}Y^0_R} &= \frac{1}{\sqrt{2}} \left(\xi_R(t) + i\eta_R(t)\right). \end{aligned} \tag{4.3}$$

Then the operator (4.2) reduces to

$$\frac{1}{\sqrt{2}} \widetilde{\lambda} \int dt \psi^0_{yL} \eta_L \,, \tag{4.4}$$

and produces a rotation through angle $2\pi\widetilde{\lambda}$ about the ξ axis [19] on the unperturbed boundary state.

As in the case of bosonic string theory we proceed in three stages:

1. Construct the boundary state corresponding to the unperturbed compactified BCFT ($\widetilde{\lambda} = 0$) including the background magnetic field $\bar{b}$.
2. Study the effect of switching on the perturbation $\widetilde{\lambda}$ by rotating the boundary state by an angle $2\pi\widetilde{\lambda}$ about the ξ axis.
3. Take the decompactification limit by removing all the winding modes in the expression for the boundary state.

For definiteness let us consider the non-BPS D9-brane in type-IIA string theory. The corresponding boundary state in the presence of magnetic field $\bar{b}$ but in the absence of any perturbation ($\widetilde{\lambda} = 0$) is given by,

$$\begin{aligned} |\mathrm{IIA9}; \bar{b}\rangle &= |\mathrm{IIA9}; \bar{b}, +\rangle - |\mathrm{IIA9}; \bar{b}, -\rangle \,, \\ |\mathrm{IIA9}; \bar{b}, \epsilon\rangle &= |\mathrm{IIA9}; \bar{b}, \epsilon\rangle_{\mathrm{mat}} \otimes |\mathrm{IIA9}; \bar{b}, \epsilon\rangle_{\mathrm{ghost}} \,, \qquad \epsilon = \pm 1 \,, \\ |\mathrm{IIA9}; \bar{b}, \epsilon\rangle_{mat} &= N_{\bar{b}} \exp\Bigg[- \sum_{n=1}^{\infty} \frac{1}{n} \left(\sigma^0_{-n} \bar{\tau}^0_{-n} + \sigma^1_{-n} \bar{\tau}^1_{-n} + \sum_{j=2}^{9} \alpha^j_{-n} \bar{\alpha}^j_{-n} \right) - \\ &\qquad - i\epsilon \sum_{r=1/2}^{\infty} \left(\chi^0_{-r} \bar{\delta}^0_{-r} + \chi^1_{-r} \bar{\delta}^1_{-r} + \sum_{j=2}^{9} \psi^j_{-r} \bar{\psi}^j_{-r} \right) \Bigg] |\mathrm{IIA9}; \bar{b}\rangle_0 \,, \\ |\mathrm{IIA9}; \bar{b}\rangle_0 &= \sum_{m \in \mathbb{Z}} \exp\left[-i\sqrt{2} m (y^0_L - y^0_R) - i\sqrt{2} m \bar{e} y^1 \right] |0\rangle \,, \end{aligned}$$

$$
\begin{aligned}
|\mathrm{IIA9};\bar{b},\epsilon\rangle_{\rm ghost} &= \exp\Big[-\sum_{n=1}^{\infty}\Big(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n}+ \\
&\qquad\qquad + i\epsilon\Big(\bar{\beta}_{-n-1/2}\gamma_{-n-1/2}-\beta_{-n-1/2}\bar{\gamma}_{-n-1/2}\Big)\Big)\Big]|\Omega\rangle\,, \\
|\Omega\rangle &= (c_0+c_0)c_1\bar{c}_1 e^{-\phi(0)}e^{-\bar{\phi}(0)}|0\rangle\,, \\
N_{\bar{b}} &= iK\frac{\mathcal{T}_9}{2}\left(1+\bar{e}^2\right)^{1/2}, \qquad (4.5)
\end{aligned}
$$

where $\mathcal{T}_9$ is the non-BPS D9-brane tension,[13] K is a convention dependent numerical factor which does not affect the final result for the sources, σ_n's are oscillators of Y, $\bar{\tau}_n$'s are defined in (3.31), $\{\chi_r,\bar{\chi}_r\}$ and $\{\psi_r,\bar{\psi}_r\}$ are the sets of oscillators of $\{\psi_y,\bar{\psi}_y\}$ and $\{\psi_x,\bar{\psi}_x\}$ respectively, and

$$\bar{\delta}_r = M_Y\bar{\chi}_r\,. \qquad (4.6)$$

From eqs. (4.5) it is clear that the overall normalisation which is the inner product between the NS-NS ground state corresponding to the identity operator and the boundary state is $N_{\bar{b}}$. This remains the overall normalisation for the rolling tachyon boundary state as the rotation keeps the NS-NS vacuum invariant. Since the effect of the magnetic field has been diagonalized to identity by dealing with the oscillators $\bar{\delta}_r$, the state in (4.5) looks, at least algebraically, exactly like a Neumann boundary state which was considered in [19]. The only difference is an extra factor of y^1-momentum dependence for each y^0 winding state.

As discussed already, the effect of the perturbation (4.1) is an SO(3) rotation about the ξ-axis by an angle $2\pi\widetilde{\lambda}$ on the boundary state. The effect of this rotation on the unperturbed boundary state can be easily studied following [19]. Using the result $Y^0-\bar{e}Y^1=\sqrt{1+\bar{e}^2}X^0$, the result for the part of the perturbed boundary state which involves either no oscillators or the states created by the action of $\chi^0_{-1/2}\bar{\delta}^0_{-1/2}$ on pure momentum states is given by,[14]

$$N_{\bar{b}}\left[\widehat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)-i\epsilon\chi^0_{-1/2}\bar{\delta}^0_{-1/2}\;\widehat{g}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)\right]|0\rangle, \qquad (4.7)$$

where

$$
\begin{aligned}
\widehat{f}(x) &= \left[1+(-1)^n\sum_{n=1}^{\infty}(\sin(\tilde{\lambda}\pi))^{2n}\left(e^{in\sqrt{2}x}+e^{-in\sqrt{2}x}\right)\right]=f(-ix)\,, \\
\widehat{g}(x) &= 1+\cos\left(2\pi\widetilde{\lambda}\right)-\widehat{f}(x)\,, \qquad (4.8)
\end{aligned}
$$

with $f(x)$ given in (2.22) for the superstring case. Therefore the net level (1/2, 1/2) contribution to the rolling tachyon boundary state is [19],

$$K\mathcal{T}_9\left(1+\bar{e}^2\right)^{1/2}\left[\chi^0_{-1/2}\bar{\delta}^0_{-1/2}\widehat{g}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)+\left(\chi^1_{-1/2}\bar{\delta}^1_{-1/2}+\sum_{j=2}^{9}\psi^j_{-1/2}\bar{\psi}^j_{-1/2}\right)\times\right.$$

[13] Note that the above state takes the same form for the D9 – $\bar{\mathrm{D}}$9-brane system or the NS-NS part of the BPS D9-brane in type-IIB string theory. In each case $\mathcal{T}_9$ denotes the tension of the corresponding brane system.

[14] Notice that the coefficients of $\chi^0_{-1/2}\bar{\delta}^0_{-1/2}$ terms in eqs. (4.7) and (4.9) have been sign flipped with respect to the corresponding terms in eqs. (4.6) and (4.7) respectively in [19]. This is because these equations in [19] were written after performing the inverse Wick-rotation, while here we are still in the Wick-rotated theory.

$$\times \widehat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right) + \left(\bar{\beta}_{-1/2}\gamma_{-1/2} - \beta_{-1/2}\bar{\gamma}_{-1/2}\right)\widehat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)\Bigg]|\Omega\rangle\,. \quad (4.9)$$

Now if we write the above state after inverse Wick-rotation and analytic continuation (3.5) in the following form,

$$-\int d^{10}k \left[\left(\tilde{A}_{\mu\nu}(k) + \tilde{C}_{\mu\nu}(k)\right)\psi^{\mu}_{-1/2}\bar{\psi}^{\nu}_{-1/2} + \tilde{B}(k)\left(\bar{\beta}_{-1/2}\gamma_{-1/2} - \beta_{-1/2}\bar{\gamma}_{-1/2}\right)\right]|\Omega, k\rangle\,, \quad (4.10)$$

with $\tilde{A}_{\mu\nu}$ symmetric and $\tilde{C}_{\mu\nu}$ anti-symmetric, then the Fourier transforms are given by the same equations as in (3.44) and (3.44) with $\mathcal{T}_{25}$ replaced by $\mathcal{T}_9$ and $f(x)$ given by (2.22). Identifying the level $(1/2,1/2)$ part of the relation $(Q_B+\bar{Q}_B)|\mathcal{B}\rangle = 0$ with the conservation law $\partial^\mu T_{\mu\nu} = \partial^\mu S_{\mu\nu} = 0$, we arrive at the same result as (3.41). This, in turn, leads to the same equations as (3.45) with $f(x)$ given in (2.22) and $\mathcal{T}_{25}$ replaced by $\mathcal{T}_9$. Note that $\mathcal{T}_9$ in this case has to be interpreted as the net tension of whatever D-brane system we consider (see footnote 13). To compute the constants K_s and K_a one can proceed in a similar way through the Dirac-Born-Infeld action as was done in the previous section and one arrives at the same results as in (3.48). This gives the expected results for the sources as given in eq. (2.20).

5. Effective field theory

Although in section 2 we used the existence of an effective field theory in describing the dynamics of tachyon condensation, we did not commit ourselves to any particular choice of the effective action. In this section we shall discuss a specific form of the low energy effective action that reproduces the answers obtained in sections 2- 4 at late time. This action is conjectured to describe correctly the classical open string dynamics when the second and higher derivatives of the tachyon and all other gauge and massless scalar fields are small. For definiteness we shall focus our attention on non-BPS D-branes, but the results can be easily generalised to brane-antibrane system as well.

The proposed action (following [16, 11, 37, 38]) for describing the dynamics of the tachyon T and gauge fields A_μ on a D-p-brane in the presence of constant background metric $g_{\mu\nu}$, anti-symmetric tensor field $b_{\mu\nu}$ and dilaton ϕ is:

$$S = -\int d^{p+1}x\, e^{-\phi}\, V(T)\sqrt{-\det A}\,, \quad (5.1)$$

where[15]

$$V(T) \simeq e^{-\alpha T/2}\,, \qquad \text{for large } T\,, \quad (5.2)$$

[15]Note that this choice of the potential $V(T)$ is in apparent contradiction with the potential derived in boundary string field theory [41]. This paradox disappears if we note that T appearing in (5.1), (5.2) could be related to the tachyon field of boundary string field theory via a complicated field redefinition which includes derivative terms. For example, an action of the form $-\int d^{p+1}x(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi) + \ldots)$ where $\ldots$ denote terms involving higher powers of derivatives, can be transformed to an action of the form $-\int d^{p+1}x(\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + \mathrm{U}(\psi) + \ldots)$ with a different form of the potential $\mathrm{U}(\psi)$, by a field redefinition of the form $\phi = f(\psi) + g(\psi)\partial^\mu\psi\partial_\mu\psi + \ldots$ by appropriately choosing the functions f and g.

$$
\begin{aligned}
A_{\mu\nu} &= g_{\mu\nu} + b_{\mu\nu} + F_{\mu\nu} + \partial_\mu T \partial_\nu T + \partial_\mu Y^m \partial_\nu Y^m \,, &(5.3)\\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \,, &(5.4)
\end{aligned}
$$

and Y^m ($m \geq p+1$) denote coordinates transverse to the D-p-brane. $\alpha = 1$ for bosonic string theory and $\sqrt{2}$ for superstring theory.

We shall now show that this action reproduces the correct time dependence of the energy-momentum tensor $T_{\mu\nu}$, and the source $S_{\mu\nu}$ for the anti-symmetric tensor field associated with rolling tachyon solution at late time. For this we note that $T_{\mu\nu}$ and $S_{\mu\nu}$ computed from this action are given by:

$$
\begin{aligned}
T_{\mu\nu} &= -\frac{1}{2} e^{-\phi} V(T) \sqrt{-\det A} \left\{ (A^{-1})_{\mu\nu} + \left(A^{-1}\right)_{\nu\mu} \right\} ,\\
S_{\mu\nu} &= -\frac{1}{2} e^{-\phi} V(T) \sqrt{-\det A} \left\{ \left(A^{-1}\right)_{\mu\nu} - \left(A^{-1}\right)_{\nu\mu} \right\} . &(5.5)
\end{aligned}
$$

For $\phi = 0$, $A_\mu = 0$, $Y^m = 0$, background $g_{\mu\nu}$, $b_{\mu\nu}$ of the form (2.3), and spatially homogeneous tachyon field, $T_{\mu\nu} + S_{\mu\nu}$ is given by the following matrix:

$$
\frac{V(T)}{\sqrt{1 - e^2 - (\partial_0 T)^2}} \begin{pmatrix} 1 & -e & \\ e & -e^2 & \\ & & \mathbb{0}_{p-1} \end{pmatrix} - V(T) \sqrt{1 - e^2 - (\partial_0 T)^2} \begin{pmatrix} 0 & 0 & \\ 0 & 1 & \\ & & \mathbb{1}_{p-1} \end{pmatrix} . \qquad (5.6)
$$

Since $T_{00} = V(T)/\sqrt{1 - e^2 - (\partial_0 T)^2}$ must be conserved, we see that as T rolls towards ∞, $\partial_0 T$ must approach $\sqrt{1 - e^2}$. Thus in this limit:

$$
T \simeq \sqrt{1{-}e^2}\, x^0 {+} C \,, \quad V(T) \simeq e^{-\alpha C/2 - \alpha\sqrt{1-e^2}\, x^0/2} , \quad \sqrt{1 - e^2 - (\partial_0 T)^2} \simeq K e^{-\alpha\sqrt{1-e^2}\, x^0/2} , \qquad (5.7)
$$

so that,

$$
\begin{aligned}
T_{00} &= K^{-1}\, e^{-\alpha C/2} ,\\
T_{01} &= 0 ,\\
T_{11} &\simeq -e^2 K^{-1}\, e^{-\alpha C/2} - K\, e^{-\alpha C/2} e^{-\alpha\sqrt{1-e^2}\, x^0} ,\\
T_{ij} &\simeq -K\, e^{-\alpha C/2} e^{-\alpha\sqrt{1-e^2}\, x^0} \delta_{ij} , \qquad \text{for} \qquad i, j \geq 2 ,\\
S_{01} &= -e K^{-1}\, e^{-\alpha C/2} , &(5.8)
\end{aligned}
$$

with all other components of $T_{\mu\nu}$ and $S_{\mu\nu}$ being zero. This reproduces the large time behaviour of (2.20) provided we identify $K^{-1}\, e^{-\alpha C/2}$ with $(1 - e^2)^{-1/2} \mathcal{T}_p (1 + \cos(2\pi\widetilde{\lambda}))/2$ and $K e^{-\alpha C/2}$ with the coefficient of $e^{-\alpha\sqrt{1-e^2} x^0}$ in the large x^0 behaviour of $(1 - e^2)^{1/2} \times \mathcal{T}_p f(\sqrt{1 - e^2} x^0)$.

As discussed in [11, 12, 20], the dynamics of the system described by the effective action (5.1) is best described in the hamiltonian formulation. Since this has been studied in detail in these papers, we shall not discuss it here.

In the presence of the background RR p-form field $C^{(p)}$ there is an additional coupling of the form [42]:

$$
\int d^{p+1}x f(T)\, dT \wedge C^{(p)} , \qquad (5.9)
$$

where

$$f(T) \simeq e^{-T/\sqrt{2}}\,, \tag{5.10}$$

for large T [20]. The source of the RR p-form field, computed from this term, has the correct time dependence [20]. If there are background q-form RR fields $C^{(q)}$ for $q < p$ as well, then (5.9) is generalised to [43]

$$\int d^{p+1}x\, f(T)\, dT \wedge e^{b+F} \wedge C\,, \tag{5.11}$$

where

$$C = \sum_{q \leq p} C^{(q)}\,. \tag{5.12}$$

For completeness of our discussion we shall briefly review the coupling of the field theory action (5.1) to background supergravity fields following [44, 45, 16, 38]. We define:

$$\mathcal{G}_{\mu\nu} = \partial_\mu Z^M \partial_\nu Z^N E^a_M E^b_N \eta_{ab}\,, \tag{5.13}$$

where Z^M (M running over 10 bosonic and 32 fermionic coordinates) are the D-brane world-volume fields describing superspace coordinates, ∂_μ denotes derivative with respect to the D-brane world-volume coordinates x^μ and E^A_M (with A running over 10 bosonic and 32 fermionic coordinates) are the bulk fields denoting the supervielbeins. The index a runs over the subset of 10 bosonic indices. We can choose the static gauge where the bosonic components Z^μ of Z^M are set equal to the D-brane world-volume coordinates x^μ, but we shall not choose any specific gauge here. We also define:

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + B_{MN} \partial_\mu Z^M \partial_\nu Z^N\,, \tag{5.14}$$

$$\mathcal{A}_{\mu\nu} = \mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu} + \partial_\mu T \partial_\nu T\,, \tag{5.15}$$

and

$$\mathcal{C}^{(q)}_{\mu_1\ldots\mu_q} = C^{(q)}_{M_1\ldots M_q} \partial_{\mu_1} Z^{M_1} \ldots \partial_{\mu_q} Z^{M_q}\,, \tag{5.16}$$

where B_{MN} is the NS-NS two form field of supergravity in superspace, $C^{(q)}_{M_1\ldots M_q}$ is the RR q-form field of supergravity in superspace, and A_μ is the U(1) gauge field on the D-brane world-volume. Finally let ϕ denote the dilaton field in the bulk. Then according to the results of [16, 38, 8], the coupling of the action (5.1) to the supergravity background will be given by:

$$S = -\int d^{p+1}x\, V(T)\, e^{-\phi} \sqrt{-\det(\mathcal{A})} + \int d^{p+1}x\, f(T)\, dT \wedge \sum_{q\leq p} \mathcal{C}^{(q)} \wedge e^{\mathcal{F}}\,. \tag{5.17}$$

This completes our discussion of the coupling of the supergravity fields to the tachyon effective field theory.

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A. Divergence of the boundary state in the $e \to 1$, $\widetilde{\lambda} \to 1/2$ limit

In this section we shall discuss the behaviour of the boundary state in the limit (2.21). From equations (2.20) it is clear that the sources of all the massless closed string states are finite. But we shall see here that the sources for higher massive states diverge in this limit. We shall demonstrate this by computing the next higher level terms in the boundary state. In particular we shall focus on the state,

$$|\mathcal{B}_{\bar{b},\widetilde{\lambda}}\rangle_{Y^0,Y^1} = N_{\bar{b}} \sum_{j,m} D^j_{m,-m} |j,m,m\rangle\rangle^{(0)}_{\bar{\tau}} \otimes \exp\left[-\sum_{n>0} \frac{1}{n}\sigma^1_{-n}\bar{\tau}^1_{-n} - 2im\bar{e}y^1\right]|0\rangle\,, \quad (A.1)$$

which is the part of $|\mathcal{B}_{\bar{b},\widetilde{\lambda}}\rangle$ given in (3.38) involving Y^0 and Y^1 . The level $(2,2)$ oscillator part of this state is given by,

$$|\mathcal{B}_{\bar{b},\widetilde{\lambda}}\rangle^{(2)}_{Y^0,Y^1} = N_{\bar{b}}\left[|\mathcal{B}_2\rangle - \sigma^1_{-1}\bar{\tau}^1_{-1}|\mathcal{B}_1\rangle - \frac{1}{2}\left(\sigma^1_{-2}\bar{\tau}^1_{-2} - (\sigma^1_{-1}\bar{\tau}^1_{-1})^2\right)|\mathcal{B}_0\rangle\right]\,, \quad (A.2)$$

where the states $|\mathcal{B}_2\rangle$, $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_0\rangle$ are the oscillator level (2,2), (1,1) and (0,0) parts respectively of the state,

$$\begin{aligned} |\mathcal{B}\rangle &= \sum_{j,m} |D;j,m\rangle \otimes \exp\left[-2im\bar{e}y^1\right]|0\rangle\,, \\ |D;j,m\rangle &\equiv D^j_{m,-m}|j,m,m\rangle\rangle^{(0)}_{\bar{\tau}}\,. \end{aligned} \quad (A.3)$$

The oscillator contribution comes only from $|D;j,m\rangle$. Let us first consider the state $|\mathcal{B}_2\rangle$. To isolate the relevant parts, we note that $|j,m,m\rangle$ has conformal weight (j^2,j^2), of which (m^2,m^2) comes from the Y^0 momentum and (j^2-m^2, j^2-m^2) comes from the oscillators. From this we see that the contribution to $|\mathcal{B}_2\rangle$ comes from three different types of terms.

1. Level (2,2) secondary in $|D;j,\pm j\rangle$. This contribution can be further divided into three different parts.

$$j \geq 1: \quad \frac{1}{2}(-\sin(\widetilde{\lambda}\pi))^{2|j|}\{(\sigma^0_{-1})^2(\bar{\tau}^0_{-1})^2 + \sigma^0_{-2}\bar{\tau}^0_{-2}\}\exp(\pm 2ijY^0(0))|0\rangle\,,$$

$$
\begin{aligned}
j = \frac{1}{2}: &\quad (-\sin(\widetilde{\lambda}\pi))\,\frac{1}{6}\,\{(\sigma^0_{-1})^2 \pm \sqrt{2}\,\sigma^0_{-2}\}\,\{(\bar{\tau}^0_{-1})^2 \pm \sqrt{2}\,\bar{\tau}^0_{-2}\}\exp(\pm iY^0(0))|0\rangle\,, \\
j = 0: &\quad \frac{1}{2}\,(\sigma^0_{-1})^2(\bar{\tau}^0_{-1})^2|0\rangle\,.
\end{aligned} \tag{A.4}
$$

2. Level (1,1) secondary in $|D;1,0\rangle$. This is given by:

$$
-\frac{1}{2}\cos(2\widetilde{\lambda}\pi)\,\sigma^0_{-2}\bar{\tau}^0_{-2})|0\rangle\,. \tag{A.5}
$$

3. The primary state in $|D;\frac{3}{2},\pm\frac{1}{2}\rangle$. This contribution is given by:

$$
\sin(\widetilde{\lambda}\pi)(\cos(2\widetilde{\lambda}\pi)+\cos^2(\widetilde{\lambda}\pi))\,\frac{1}{6}\{\sqrt{2}(\sigma^0_{-1})^2 \mp \sigma^0_{-2}\}\{\sqrt{2}(\bar{\tau}^0_{-1})^2 \mp \bar{\tau}^0_{-2}\}\exp(\pm iY^0(0))|0\rangle\,. \tag{A.6}
$$

Note that whereas the phases of the states $|D;j,\pm j\rangle$ and $|D;1,0\rangle$ were determined in [18, 19], the phase of $|D;\frac{3}{2},\pm\frac{1}{2}\rangle$ needs to be determined afresh by requiring that at $\widetilde{\lambda}=\frac{1}{2}$ the boundary state reduces to the known boundary state. This will be checked explicitly later.

Combining these results together, we get the following final expression for $|\mathcal{B}_2\rangle$,

$$
\begin{aligned}
|\mathcal{B}_2\rangle = \Bigg[&\sigma^0_{-2}\bar{\tau}^0_{-2}B^{(2)}\left(\widetilde{\lambda};X^0(0)\right) + (\sigma^0_{-1})^2(\bar{\tau}^0_{-1})^2B^{(4)}\left(\widetilde{\lambda};X^0(0)\right) \\
&+ \left(\sigma^0_{-2}(\bar{\tau}^0_{-1})^2 + (\sigma^0_{-1})^2\bar{\tau}^0_{-2}\right)B^{(3)}\left(\widetilde{\lambda};X^0(0)\right)\Bigg]|0\rangle\,,
\end{aligned} \tag{A.7}
$$

where,

$$
\begin{aligned}
B^{(2)}\left(\widetilde{\lambda};X^0(0)\right) &= -\frac{1}{2}\cos(2\pi\widetilde{\lambda}) + \sin(\pi\widetilde{\lambda})\cos^2(\pi\widetilde{\lambda})\cos\left(\sqrt{1+\bar{e}^2}X^0(0)\right) + \\
&\quad +\frac{1}{2}\left(\widehat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)-1\right), \\
B^{(3)}\left(\widetilde{\lambda};X^0(0)\right) &= -i\sqrt{2}\sin(\pi\widetilde{\lambda})\cos^2(\pi\widetilde{\lambda})\sin\left(\sqrt{1+\bar{e}^2}X(^00)\right), \\
B^{(4)}\left(\widetilde{\lambda};X^0(0)\right) &= 2\sin(\pi\widetilde{\lambda})\cos^2(\pi\widetilde{\lambda})\cos\left(\sqrt{1+\bar{e}^2}X^0(0)\right) + \frac{1}{2}\widehat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right),
\end{aligned} \tag{A.8}
$$

where the functions $\widehat{f}(x)$, $\widehat{g}(x)$ were defined in eq. (3.43).

The results for $|\mathcal{B}_1\rangle$ and $|\mathcal{B}_0\rangle$ are already known from the analysis of [19] which was also discussed in section 3.5. These are given by,

$$
|\mathcal{B}_1\rangle = -\sigma^0_{-1}\bar{\tau}^0_{-1}\widehat{g}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)|0\rangle\,, \qquad |\mathcal{B}_0\rangle = \widehat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)|0\rangle\,. \tag{A.9}
$$

Collecting all these results for $|\mathcal{B}_2\rangle, |\mathcal{B}_1\rangle$ and $|\mathcal{B}_0\rangle$ and substituting them in eq. (A.2) one finally gets,

$$
\begin{aligned}
|\mathcal{B}_{\bar{b},\widetilde{\lambda}}\rangle^{(2)}_{Y^0,Y^1} = N_{\bar{b}}\Bigg[&\sigma^0_{-2}\bar{\tau}^0_{-2}\left\{\sin\left(\pi\widetilde{\lambda}\right)\cos^2\left(\pi\widetilde{\lambda}\right)\cos\left(\sqrt{1+\bar{e}^2}X^0(0)\right) - \frac{1}{2}\widehat{g}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)\right\} + \\
&+(\sigma^0_{-1}\bar{\tau}^0_{-1})^2\left\{2\sin\left(\pi\widetilde{\lambda}\right)\cos^2\left(\pi\widetilde{\lambda}\right)\cos\left(\sqrt{1+\bar{e}^2}X^0(0)\right) + \frac{1}{2}\widehat{f}\left(\sqrt{1+\bar{e}^2}X^0(0)\right)\right\} -
\end{aligned}
$$

$$-i\sqrt{2}\left(\sigma^0_{-2}(\bar\tau^0_{-1})^2+(\sigma^0_{-1})^2\bar\tau^0_{-2}\right)\sin(\pi\widetilde\lambda)\cos^2(\pi\widetilde\lambda)\sin\left(\sqrt{1+\bar e^2}X^0(0)\right)+$$
$$+\sigma^0_{-1}\sigma^1_{-1}\bar\tau^0_{-1}\bar\tau^1_{-1}\,\widehat g\left(\sqrt{1+\bar e^2}X^0(0)\right)-$$
$$-\frac{1}{2}\left(\sigma^1_{-2}\bar\tau^1_{-2}-\left(\sigma^1_{-1}\bar\tau^1_{-1}\right)^2\right)\widehat f\left(\sqrt{1+\bar e^2}X^0(0)\right)\Bigg]|0\rangle\,, \tag{A.10}$$

One can now use the matrices v and M_Y given in equations (3.10) and (3.24) to express (A.10) in terms of the α and $\bar\alpha$ oscillators. To exhibit the divergence of the boundary state we shall state the result for the coefficient of the state $(\alpha^0_{-1}\bar\alpha^0_{-1})^2|0\rangle$. This is given by,

$$K\mathcal{T}_p(1+\bar e^2)^{-3/2}\left[\left(1+\cos\left(2\pi\widetilde\lambda\right)\right)\left(\sin(\pi\widetilde\lambda)\cos\left(\sqrt{1+\bar e^2}x^0\right)-\bar e^2\right)+\frac{1}{2}(1+\bar e^2)^2\widehat f\left(\sqrt{1+\bar e^2}x^0\right)\right]. \tag{A.11}$$

After inverse Wick-rotation and the analytic continuation (3.5) this becomes,

$$K\mathcal{T}_p(1-e^2)^{-3/2}\left[\left(1+\cos\left(2\pi\widetilde\lambda\right)\right)\left(\sin\left(\pi\widetilde\lambda\right)\cosh\left(\sqrt{1-e^2}x^0\right)+e^2\right)+\frac{1}{2}(1-e^2)^2 f\left(\sqrt{1-e^2}x^0\right)\right], \tag{A.12}$$

In the limit (2.21), the first term in the above expression diverges while the second term goes to zero. The first term diverges as $\lim_{e\to1}4K\rho(1-e^2)^{-1}$, where $\rho=\frac{\mathcal{T}_p}{2}(1-e^2)^{-1/2}(1+\cos(2\pi\widetilde\lambda))$ is the energy density T^{00} in this limit and it is finite. We can also verify that at $\widetilde\lambda=1/2$, $e<1$, the state (A.12) vanishes identically, as is required by the fact that at finite e, $\widetilde\lambda=1/2$ represents the tachyon vacuum configuration.

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JHEP11(2002)047